Measuring Ethnic Clustering and Exposure with the Q statistic: An Exploratory Analysis of Irish, Germans, and Yankees in 1880 Newark

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Abstract. The study of population patterns has animated a large body of urban social research over the years. An important part of this literature is concerned with the identification and measurement of segregation patterns. Recently, emphatic calls have been made to develop measures that are better able to capture the geography of population patterns. The objective of this paper is to demonstrate the application of the *Q* statistic, developed for the analysis of spatial association of qualitative variables, to the detection of ethnic clustering and exposure patterns. The application is to historical data from 1880 Newark in the United States, with individuals classified by ethnicity and geo-coded by place of residence. Three ethnic groups, termed Irish, Germans, and Yankees are considered. Exploratory analysis with the *Q* statistic identifies significant differences in the tendency of individuals and building occupancy to cluster by ethnicity. In particular, there is evidence of a strong affinity within ethnic clusters, and some intermingling between Yankee and Irish residents. In contrast, the exposure of Germans to individuals of other groups is found to be more limited.

Keywords. Ethnicity, segregation, clustering, exposure, spatial association, *Q* statistic

Introduction and Context for Research

Population segregation is not a new phenomenon. More than a century ago, in 1903, DuBois saw it as a barrier to comity between ethnic groups, and lamented that it "caused each to see the worst in the other" (DuBois, 1903; cited in Charles, 2003). In all probability, segregation was old even then.

There are a few modern accounts of historical segregation patterns and their effects. Kantrowitz (1979), for example, studied the segregation of minority populations in Boston from 1830 to 1970 as a way to inform current (at the time) debates on public school desegregation programs. Boyd (1998) investigated the situation of black merchants in the early 1900s in United States, and suggested that, whatever other social effects it may have had, segregation seemed to have encouraged the emergence of a new class of black entrepreneurs. In an example of how segregation can emerge and be perpetuated, Gotham (2000) traced the origins of residential segregation in Kansas City in the first half of the twentieth century, back to the racial attitudes of key players in the budding real estate market in that city. These modern studies and others (e.g. Hershberg et al, 1979; Spain, 1979) provide valuable historical perspectives on the phenomenon. Besides some epochal accounts of anecdotal value (e.g. such as by DuBois), it appears that the formal study of segregation only started with the studies of the Chicago School of Sociology that empirically described the social ecology of Chicago neighborhoods (Dawkins et al, 2007). Population segregation research has since gained in scope and depth, and today it is a topic that animates a large body of urban social research from a number of different perspectives, including sociology (e.g. Logan and Zhang, 2010), geography (e.g. Deurloo and de Vos, 2008), urban studies (e.g. Harsman, 2006), and economics (e.g. Cutler et al, 2008), to mention just a few.

One of the reasons why segregation is of interest is that it remains a key to understanding inequality and social mobility issues, and therefore is still of significant social science and policy interest (Pettigrew, 1979; Charles, 2003; Simpson, 2004). The specific focus of segregation research varies by context, for instance from ethnicity and race in the US (Massey and Denton, 1993), the UK (Peach, 1996; Johnston et al, 2002), and Australia (Poulsen and Johnston, 2000), to religion in Northern Ireland (Lloyd, 2010), income and race in Brazil (Feitosa et al, 2007), and age and income Canada (Smith, 1998; Fong and Shibuya, 2000). The general motivation, however, remains the same: trying to understand the processes and patterns of separation, whether willing or

imposed, of members of a social group from others. While interest in segregation seems to have ebbed and flowed in the past few decades (Charles, 2003), judging from the number of papers, specially collected issues (e.g. Kaplan and Woodhouse, 2004; Kaplan and Woodhouse, 2005; Wong et al, 2007; Dawkins et al, 2007; Simpson and Peach, 2009; Bolt et al, 2010), and the passion that animates some of the debates (e.g. Peach, 2009), segregation research is currently at a high point, and work continues along numerous fronts.

One area of ongoing interest in the segregation literature is motivated by the need to produce reliable statistics to inform academic and policy discussions. Use of the Dissimilarity Index was for long the standard approach used in segregation studies (Massey and Denton, 1988). Especially after the systematic review of concepts and measures of Massey and Denton (1988), it became generally recognized that segregation is a concept that spans multiple dimensions, not easily captured by any one single index. This prompted research that developed a number of indicators useful to capture the various dimensions of segregation, including Theil's Index, the delta index, the Gini Index, etc.

One limitation of many early indicators used to measure the different dimensions of segregation is that they consider people in space but rarely their spatial relationships beyond propinquity in the same administrative division (e.g. the census tract). In other words, many of these indices operate by aggregating areal population values while disregarding all other spatial structures (White, 1983; Reardon and O'Sullivan, 2004). In recent years, increasingly emphatic calls have been made to develop measures that are better able to capture geographic patterns of segregation (Wong, 1997; Wong, 1999; Brown and Chung, 2006; Johnston et al, 2009). A geographical perspective adds depth to segregation analysis by allowing researchers to consider the spatial association of segregation measures (Wong, 1997; Dawkins, 2004), the local patterns of segregation (Wong, 2002; Feitosa et al, 2007; Lloyd, 2010), and importantly from our perspective, by conceptualizing spatial association itself as a measure of segregation (Wong, 1999; Brown and Chung, 2006; Johnston et al, 2009).

The objective of this paper is to demonstrate the use for the analysis of population segregation of the newly developed *Q* statistic for spatial association of qualitative variables (Ruiz et al, 2010). As will be shown, the *Q* statistic can be used to assess patterns of clustering and exposure. It constitutes a valuable tool not only to explore these types of patterns, but also to statistically test them against the hypothesis

of spatial randomness, an old debate in the literature (see Reiner, 1972; and Zelder, 1972). Unlike other approaches that are exclusively area-based and designed for continuous variables, the support of *Q* can also be the point and is designed for qualitative variables. This means that it can be applied to analysis at the personal level, with, say, ethnicity defining a qualitative attribute of the individual. Furthermore, it can also be scaled up to other levels of geography by categorizing higher level outcomes.

We demonstrate the proposed approach by means of historical data from 1880 Newark in the US, with individuals classified by ethnicity and marital status, and geo-coded by place of residence. Three ethnic groups, termed Yankees, Irish, and Germans are considered. Application of the *Q* statistic identifies significant differences in the tendency of individuals to cluster by ethnicity, and of buildings by dominant occupancy. In particular, there is evidence of a strong affinity for clustering within ethnic groups, and some intermingling between Yankee and Irish residents. In contrast, exposure between German individuals and members of the other groups is significantly more limited. The same is observed for predominantly German and other buildings.

The structure of the paper is as follows: in the following section we briefly review previous work that has adopted a spatially-explicit perspective to the measurement of population patterns; this is followed by a technical section that describes the *Q* statistic. Next, we introduce the dataset used in the application, followed by the results of the analysis. Finally, in the concluding section we summarize our main points and sketch directions for future research.

Literature Review: Measuring Segregation Spatially

A number of recent papers in the literature discuss the state of the practice, the art, and challenges in segregation research (e.g. Kaplan and Woodhouse, 2004; Kaplan and Woodhouse, 2005; Wong et al, 2007; Dawkins et al, 2007; Simpson and Peach, 2009; Bolt et al, 2010). Readers interested in a more extensive panoramic of the field are redirected to these manuscripts. In our review we concentrate only on recent contributions that discuss the spatial aspects of measuring segregation. These studies tend to emphasize one or a combination of three major issues. First, traditional measures are notorious for their inability to appropriately incorporate the spatial relationships between units of analysis (i.e. the "checkerboard problem" of White, 1983). The second issue is the presence of spatial patterns in the segregation measures themselves. Finally, there are the questions of aggregation and scale, which may have an important impact

on findings and recommendations. These issues are not necessarily independent, but, as a number of papers reviewed below show, may in fact interact in various ways.

An early criticism of the standard tool of segregation research, the Dissimilarity Index, came from White (1983), who noted that paring geography out renders the measure insensitive to spatial pattern, and incapable of distinguishing between residential clusters and ghettos. This issue, shared by most other segregation measures, is the so-called checkerboard problem. In an attempt to improve on this state of affairs, White proposed a proximity index that was directly based on distance between members of the population. White's index, unfortunately, is difficult to interpret, although several of its base components (i.e. the average distance between members of the same or different population groups) do indeed provide valuable spatial information. A notable aspect of White's proximity index is that it incorporated, perhaps for the first time in segregation research, a distance-decay function. This function is in essence a spatial kernel used to decrease the contribution to the proximity index of a given pair of individuals as the distance between them increases (earlier Jakubs, 1981, used the distances between all pairs of areal units for optimization). Kernel functions have since taken on a prominent role in spatial segregation measures (see for instance Wong, 1998, who used a rectangular kernel and alluded to distance-decay functions).

Kernel functions directly speak to issues of the spatial relationships and scale. Reardon and O'Sullivan (2004) proposed a formal framework to generate spatial segregation measures. At the core of this framework is the use of kernel functions to establish relations of proximity. Depending on data availability, the measures proposed are applicable at very high levels of granularity, potentially even the individual person. Despite this, the measures are based on population proportions and densities, and are therefore inherently areal – however, the areas can be established by the analyst as part of defining the functional form and parameters of the kernel function. Reardon and O'Sullivan suggested that analysts can specify these elements of the framework based on theoretical notions of how space influences social interaction. In practice, it has been more common to use kernel-based approaches in an exploratory fashion, to investigate separate but related issues of scale and aggregation that form part of the modifiable areal unit problem (Wong et al, 1999). Feitosa et al. (2007), for instance, demonstrate their global and local indicators of segregation by exploring bandwidths ranging between 400 m to 4400 m. O'Sullivan and Wong (2007) proposed to use the union and

intersection of two kernel functions for different population groups to measure segregation. The approach was applied in their paper to the cities of Philadelphia and Washington, D.C., using kernel bandwidths of 2.5 to 10 km, in 2.5 km increments. Reardon et al. (2008) implemented the principles outlined in Reardon and O'Sullivan (2004) to investigate the scale of segregation using bandwidths from 100 m to 4,000 m. This produces so-called "segregation profiles" that track the degree of segregation at different scales. A similar idea is put to work in a paper by Deurloos and de Vos (2008), who applied the *k*-function-inspired multi-scale measure of Marcon and Puech (2003) to assess the concentration of various ethnic groups with respect to each other, at relatively small scales up to 560 m. More recently, Lloyd (2010) used geographically weighted descriptive statistics to investigate population concentration patterns by community background Northern Ireland.

The ability to investigate segregation patterns at various scales begs the question of whether it makes a difference, and research has been conducted to clarify this, by comparing the results of conventional (census geography-based) to spatial measures of segregation. Kramer et al. (2010) investigated black and white segregation in 231 of the largest Metropolitan Statistical Areas in the US using the diversity and exposure indices, and, after Reardon and O'Sullivan (2004), surface-based versions of the same. The results indicate that both types of measures, spatial and aspatial, are highly correlated but the differences are not uniform, a fact that may mask potentially valuable information. In particular, these researchers report that the differences between census tract-based and surface-based measurements are greater for smaller cities and at higher levels of resolution (i.e. when calculations are made based on smaller kernel bandwidths). The latter results stands in contrast to an earlier report by Wong (1997) indicating that the dissimilarity index tends to be deflated at lower levels of resolution (i.e. when calculations are based on geographically larger units of analysis). While Wong (1997) does not report results by population size, he argues that the dissimilarity index is sensitive to the level of autocorrelation of the population values. These values, unfortunately, are not reported by Kramer et al. (2010). Other research by Dawkins (2004) does in fact confirm that spatial autocorrelation can in some cases account for a large part of the measured segregation.

That spatial autocorrelation provides a naturally spatial measure of segregation may seem evident. Already, Massey and Denton (1988) mentioned the use of measures of spatial autocorrelation to assess clustering patterns. However, besides work by Wong

(1999) that propounded the use of centrographic analysis as a way to assess segregation levels, until recently there were only few examples of research where autocorrelation measures were used as measures of segregation. In part, this may have been due to the fact, noted by Dawkins (2004), that autocorrelation measures, such as Moran's Coefficient, are limited to the analysis of clustering and fail to capture unevenness (p. 835). With the advent of local spatial analysis techniques, in particular the G_i and LISA statistics (Getis and Ord, 1992; Anselin, 1995), there are increased opportunities to investigate clustering and unevenness. A few recent papers adopt this approach. These include Logan et al. (2002) and the use of LISA statistics to identify immigrant enclaves and ethnic communities in New York and Los Angeles. Brown and Chung (2006) advocated a geographical perspective in the analysis of segregation, and supported their case with a study of Franklin County, Ohio. There, it was shown that blacks tend to be more clustered spatially than whites, and that Asians and Hispanics, while displaying significant levels of clustering, do not reach the same levels as blacks and whites. In addition to the analysis of clustering using Moran's Coefficient, the local version of the statistic detected patterns of unevenness, with black/white over/underrepresentation in the central city respectively, and the opposite pattern for the suburbs. The typical central/suburban dichotomy, in contrast, did not hold for Asians, a group that is overrepresented in the northwest and underrepresented in the southern part of the city. Three distinctive clusters of Hispanic residents were also found. Even more recently, Johnston et al. (2009) also made a plea to "put more geography in" and showed the way by means of global and local autocorrelation analysis of population patterns in Auckland, New Zealand. The results of this analysis indicated not only high levels of population concentration for Europeans, Maori, Pacific Islanders, and Asians, but also the regions of the city where these concentrations are particularly marked.

Together, the works reviewed here persuasively show the richness of detail that can be achieved by adopting a geographical perspective. In what follows, we describe an alternative analytical framework based on the use of the *Q* statistic

Methods: Spatial Association of Qualitative Variables

Autocorrelation analysis of continuous variables is a time-honored practice in analytical geography (Getis, 2008). More recently, interest in the analysis of variables of a qualitative/nominal nature has spurred renewed attention on techniques useful to explore and model spatial qualitative processes. One recent development is *Q*, a statistic

designed to test the spatial association of qualitative variables (Ruiz et al, 2010). As we show below, *Q* provides an intuitive way to measure ethnic clustering and exposure patterns. In this section we briefly discuss the conceptual basis for the use of Q. Further, we provide a brief description of the statistic (additional technical details can be found in the paper by Ruiz et al.), and introduce two technical refinements that are useful for our analytic framework.

Conceptual Basis

In their original review, Massey and Denton (1988) proposed five dimensions of segregation, namely *eveness* (the over- or under-representation of a social group in specific areas), *exposure* (of members of one group to members of other groups), *concentration* (of members of a group in an area), *centralization* (concentration in the central city), and *clustering* (the extent to which members of a group adjoin one another). This typology of segregation has been revisited by later authors. It has been argued, for instance, that the relevance of centralization is much diminished in contemporary polynucleated cities. This has led to a reduction in the number of dimensions of segregation. Further, the remaining dimensions are seen to lie in a two dimensional continuum: eveness-clustering and isolation-exposure in the case of Reardon and O'Sullivan (2004), and eveness-concentration and clustering-exposure in the case of Brown and Chung (2006). The original work of Massey and Denton (1988; see tables 4 and 5) already indicated the interrelationships between some of these dimensions, in particular clustering and exposure, but also shows that eveness and clustering measures tend to retain a fairly distinctive character.

As should become clear below, the *Q* statistic more naturally fits the clustering-exposure dimension of Brown and Chung. According to these authors, the dimension of clustering refers to units close to others units of the same type, thus forming a contiguous grouping of likes. Further, the dimension of exposure is the degree to which units share a neighborhood with other units of different types. It follows then that high clustering is in fact a manifestation of low exposure and viceversa (Brown and Chung, 2006, p. 126). The *Q* statistic is built around proximity relationships of spatial units that are classified according to their type. By designing neighborhoods of a specified size, it becomes possible to summarize the class membership of all units in a given neighborhood, and therefore to investigate to what extent the members are of the same type (clustering) or of different types (exposure).

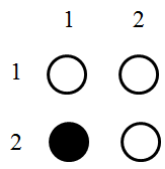
This basic notion is formalized next.

Q *Statistic*

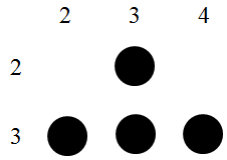
The statistic is developed for the analysis of a spatial variable, say **Y**, that is the outcome of a discrete process. In other words, each realization *y* of the process can take one and only one of *k* different values, say a_1, a_2,..., a_k, that are recorded at sites i= 1, 2,..., N with coordinates $\boldsymbol{s}_i$. In the simplest case, when k=2, the process can be represented by a black-and-white map (i.e. a_1=w=0 and a_2=b=1). Borrowing a set of typical diagrams from the segregation literature, maps with different spatial configurations of the qualitative variable could be as shown in Figure 1. Note that for simplicity, the diagrams use a regular distribution of cases; in actual practice, the statistic can be applied to an irregular distribution of cases as well. The diagrams represent two extreme cases of non-random patterns, as well as one random pattern.

In order to capture relations of proximity between realizations of the spatial variable, we define for a location $\boldsymbol{s}_0$ a local neighborhood of size *m*, called an *m*-surrounding. While the size of the *m*-surrounding is determined by the analyst (the analog of defining the pattern of contiguities in matrix **W** in autocorrelation analysis), for the sake of the example consider *m*-surroundings of size 4 (m=4), to give subsets of 4 cells. A rule must be defined to identify the *m*-1 nearest neighbors that, together with site $\boldsymbol{s}_0$, form neighborhood of size 4. Ruiz et al. (2010) propose taking the *m*-1 nearest neighbors based on distance, and in the case of ties, based on the smallest angle (counterclockwise) from the *x* axis to ensure the uniqueness of each member in the *m*-surrounding.

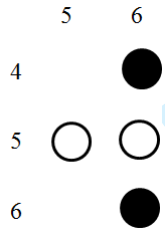
According to these rules, we can find the values of the spatial variable in the 4-surrounding for specific locations; for instance, for the third diagram in the figure, the 4-surrounding of the cell in the first row and first column, or site (1,1), is:



Clearly, this white unit displays high clustering, and low exposure to black. The 4-surrounding for site (3,3) is:



The black unit in (3,3) is part of a cluster of black, and has zero exposure to white. As a final example, consider the 4-surrounding for site (5,6):



As it can be seen, in this case the white unit has equal clustering and exposure properties.

The shapes of the *m*-surroundings in these examples are different due to the rules used to select the *m*-1 nearest neighbors. Since the arrangement of cases is regular, there are distance ties that are broken by making reference to the angle with respect to site $\boldsymbol{s}_0$. These rules are used for convenience, but can be modified to incorporate anisotropy or other considerations. Distance ties are extremely rare when the distribution of cases is not regular, and the *m*-surroundings will in general be irregularly shaped.

The local configuration of values of *y* can be represented in a compact form by means of *symbols*. A symbol, denoted by σ, is a string that collects in a pre-specified order the values of the variable in an *m*-surrounding. According to our rule, the symbol for site (1,1) in the third diagram is {w,w,b,w}={0,0,1,0}. The first element of the string is the value of *y* at $\boldsymbol{s}_0$, that is, at site (1,1). Cells (1,2) and (2,1) are equidistant from (1,1), and therefore the tie is broken by making reference to the angle from the *x* axis, so that (1,2) is picked first. Cell (2,2) is picked last because it is the farthest of *m*-1 nearest neighbors. In similar fashion, the symbol for site (3,3) becomes {b,b,b,b}={1,1,1,1}, and the symbol for site (5,6) becomes {w,b,w,b}={0,1,0,1}. Every other cell in the diagram can be symbolized in the same way.

Now, since there are *k*=2 classes and the *m*-surrounding is of size 4, it is straightforward to see that there are in fact k^m=16 unique symbols, as shown in Table 1. As per the table, we say that location (1,1) in the third diagram is of type σ_3, location (3,3) is of symbol σ_{16}, location (5,6) of symbol σ_9, and so on. After symbolizing the

locations, it is possible to calculate the frequency of each symbol as the number of locations that are of type σ_j:

$$n_{\sigma_j} = \#\left(\boldsymbol{s} \mid s \text{ is of type } \sigma_j\right) \tag{1}$$

The relative frequency is simply the number of times that symbol σ_j (j = 1, 2,..., k^m) is observed, divided by the number of symbolized locations S . It should be clear that there will be some overlap between m-surroundings at different locations. This overlap may compromise some approximations required for developing a test of hypothesis, and so in order to reduce the overlap, the number of symbolized locations in general is not the same as the number of observations N (more on this below). The relative frequency of each symbol is then:

$$p_{\sigma_j} = \frac{n_{\sigma_j}}{S} \tag{2}$$

The frequencies and relative frequencies of the symbols (ignoring the overlap condition) in each of the three examples in Figure 1 are shown in Table 2. It can be seen there that in general when the map is patterned, a small number of symbols tends to dominate, whereas when the map is random no single symbol dominates. For a fixed m≥2, the relative frequency of symbols can be used to define the *symbolic entropy* of the spatial process as the Shanon's entropy of the distinct symbols:

$$h(m) = -\sum_j p_{\sigma_j} \ln(p_{\sigma_j}) \tag{3}$$

When a sequence of values is repeated in space, the information content of the map will in general be low since symbols become to some extent predictable (as is the case of maps with strong patterns of spatial association). Mathematically, the symbolic entropy tends in this case to 0 because $p_{\sigma_i} \to 1$ and $p_{\sigma_j} \to 0$ for all $j \neq i$, which implies that $p_{\sigma_i} \ln(p_{\sigma_i}) \to 0$ and $p_{\sigma_j} \ln(p_{\sigma_j}) \to 0$. In the particular case when the values of the variable appear with identical frequency (in the example black and white appear 18 times each) the expected relative frequency for a random spatial process is $p_{\sigma_j} = 1/k^m$ for all j. Therefore, the entropy function in this case is bounded between $0 < h(m) \leq \ln\left(k^m\right)$. The Q statistic is essentially a likelihood ratio test between the symbolic entropy of the observed pattern, and the entropy of the system under the null

hypothesis of a random spatial sequence:

$$Q(m) = 2S\left(\ln\left(k^m\right) - h(m)\right) \tag{4}$$

The statistic is asymptotically χ^2 distributed with k^m-1 degrees of freedom. Let $0 \le \alpha \le 1$. A decision rule to reject the null hypothesis of spatial randomness at a confidence level of 100(1-α)% can be established as follows:

$$\begin{cases} \text{If } Q(m) > \chi^2_{k^m - 1\alpha} \text{ then reject } H_0 \\ \text{Otherwise do not reject } H_0 \end{cases} \tag{5}$$

where $P(\chi^2_{k^m-1} > \chi^2_{k^m-1\alpha}) = \alpha$.

In general, the frequencies of the outcomes a_j are not identical (e.g. some ethnic groups are minorities). In such cases, the upper bound of the entropy function (for a spatial random sequence) depends on the frequency of the various outcomes a_j, and is given by:

$$-\sum_{i=1}^{k^m} \frac{n_{\sigma_i}}{S} \sum_{j=1}^{k} \alpha_{ij} \ln\left(q_j\right) \tag{6}$$

where α_{ij} is the number of times that class a_j appears in symbol σ_i and $q_j = P(y = a_j)$. The Q statistic then becomes:

$$Q(m) = 2S\left(\sum_{i=1}^{k^m} \frac{n_{\sigma_i}}{S} \sum_{j=1}^{k} \alpha_{ij} \ln\left(q_j\right) - h(m)\right) \tag{7}$$

which is also asymptotically $\chi^2_{k^m-1}$ distributed and uses the decision rule in (5). The finite sample properties of the statistic are comprehensively explored in Ruiz et al. (2010).

As noted above, and discussed more in depth in Ruiz et al. (2010), performance of the statistic can become compromised due to the overlap of m-surroundings. In order to meet all key approximations for testing, the overlap is controlled by letting the maximum number of symbolized locations S to be less than the actual number of observations N, as follows:

$$S = \left[\frac{N - m}{m - r}\right] + 1 \tag{8}$$

where $[x]$ is the integer part of a real number x, and r is the overlap degree allowed between the m-surroundings of proximate locations. In order to select determine S

locations for the analysis, coordinates are selected such that for any two coordinates s_i, s_j the number of overlapping nearest neighbours of s_i and s_j are at most *r*. A procedure to select *S* locations that satisfy the designated overlap degree is introduced in Ruiz et al. (2010, p. 289). The set *S* is defined recursively as follows. First chose a location $\mathbf{s}_0$ at random and fix an integer r with $0 \le r < m$. Let $\{s_1^0, s_2^0, ..., s_{m-1}^0\}$ be the set of nearest neighbours to $\mathbf{s}_0$, where the s_i^0's are ordered by distance to $\mathbf{s}_0$, or angle in the case of ties. Let us call $s_1 = s_{m-r-1}^0$ and define $A_0 = \{s_0, s_1^0, ..., s_{m-r-2}^0\}$. Take the set of nearest neighbours to $\mathbf{s}_1$, namely $\{s_1^1, s_2^1, ..., s_{m-1}^1\}$, in the set of locations $S \setminus A_0$ and define $s_2 = s_{m-r-1}^1$. Now for $i > 1$ we define $s_i = s_{m-r-1}^{i-1}$ where s_{m-r-1}^{i-1} is in the set of nearest neighbours to s_{i-1}, $\{s_1^{i-1}, s_2^{i-1}, ..., s_{m-1}^{i-1}\}$, of the set $S \setminus \{\cup_{j=0}^{i-1} A_j\}$. Continue this process while there are locations to symbolize.

Simulation experiments reported in Ruiz et al. (2010) indicate that increasing the degree of overlap leads to a smaller size of the statistic (thereby reducing the risk of false positives) but at the cost of reduced power. Increasing the degree of overlap allows the analyst to retain more observations, which increases the power of the statistic, but also slightly the risk of false positives.

Equivalent Symbols

The symbolization scheme proposed by Ruiz et al. (2010) and described above – we call these *standard symbols* – contains a large amount of topological information regarding the units of analysis, including proximity and direction. In this sense, the scheme is fairly general. On the other hand, it is easy to see that the combinatorial possibilities very quickly can become unmanageable. For a process with *k*=3 outcomes and *m*=5, the number of symbols becomes 3^5=243; for *k*=6 and *m*=4 it is 6^4=1,296. Depending on the number of observations *N*, the explosion in the number of symbols can very rapidly consume degrees of freedom for hypothesis testing, since as a rule of thumb it is recommended that the number of symbolized locations be at least 5 times the number of symbols used (e.g. $S \ge 5k^m$), and *S* will usually be a fraction of *N* as per Equation (8). In addition, the large number of symbols may obfuscate the interpretation of results.

As an alternative, hereby we propose a symbolization protocol that sacrifices

some amount of topological detail for conciseness. The alternative is based on the standard scheme; however, instead of retaining proximity and direction relationships, it maintains only the total number of occurrences of each outcome in an *m*-surrounding. We call these *equivalent symbols*. Table 3 shows the equivalent symbols corresponding to the standard symbols for *k*=2 and *m*=4 (Table 1). These equivalent symbols are read as follows: σ^*_1 is a location for which, in a neighborhood of 4, there are no blacks; σ^*_2 is for locations where 3 out of 4 neighbors are white. Note the reduction in information: σ^*_2 includes the case where the nearest neighbor of a white is black (σ_4), as well as the case where the first two nearest neighbors are white (σ_2). In exchange, the number of symbols is greatly reduced, which relieves some pressure to work with smaller datasets. At least as importantly, interpretation of results also becomes more straightforward, something that facilitates the visual inspection of the frequency of classes.

Intervals of Confidence for Histogram

In addition to providing a decision rule to reject the null hypothesis of spatial randomness, the Q statistic can also be used to explore in more detail the characteristics of pattern. This is done by preparing a visual representation of the frequency or relative frequency of classes. This can be a leaf-and-stem plot (an example of which appears in Table 2) or a histogram. A question of interest is whether a specific symbol appears more or less frequently than what would be expected by chance. This question can be addressed by including as part of the histogram, the intervals of confidence with respect to the expected (relative) frequency under the null hypothesis. These intervals of confidence can be calculated in the following way.

Fix a symbol σ. Then the number of times that a symbolized location is of σ-type, namely Ψ_σ, can be approximated to a Binomial distribution:

$$\Psi_\sigma \approx B(S, p_\sigma) \tag{9}$$

where S is the total number of symbolized location. When S is large enough, the binomial distribution can be approximated to a Normal distribution with the following parameters:

$$\Psi_\sigma \approx B\left(S, p_\sigma\right) \approx N\left(Sp_\sigma, \sqrt{Sp_\sigma\left(1-p_\sigma\right)}\right) \tag{10}$$

And therefore we get that:

$$\frac{\Psi_\sigma - Sp_\sigma}{\sqrt{Sp_\sigma(1-p_\sigma)}} \approx N(0,1) \tag{11}$$

Let $0 \leq \alpha \leq 1$. Let $z_{\alpha/2}$ be the real number satisfying that $P(N(0,1) \geq z_{\alpha/2}) = \alpha/2$. Then, since the Normal standard distribution N(0,1) is symmetric with respect to $x=0$ axis, we have that:

$$\begin{aligned} \alpha &= P\left(-z_{\alpha/2} \leq \frac{\Psi_\sigma - Sp_\sigma}{\sqrt{Sp_\sigma(1-p_\sigma)}} \leq z_{\alpha/2}\right) \\ &= P\left(Sp_\sigma - z_{\alpha/2}\sqrt{Sp_\sigma(1-p_\sigma)} \leq \Psi_\sigma \leq Sp_\sigma + z_{\alpha/2}\sqrt{Sp_\sigma(1-p_\sigma)}\right) \end{aligned} \tag{12}$$

and therefore we get that:

$$\left(p_\sigma - z_{\alpha/2}\sqrt{\frac{p_\sigma(1-p_\sigma)}{S}}, p_\sigma + z_{\alpha/2}\sqrt{\frac{p_\sigma(1-p_\sigma)}{S}}\right) \tag{13}$$

is a 100(1-α)% confidence interval for the relative frequency of a symbol to occur $\frac{\Psi_\sigma}{S}$.

Case Study: Data

Data used in these analyses were compiled by the Urban Transition Historical GIS Project (UTP) at Brown University (Logan et al, 2010; see also www.s4.brown.edu/utp). The project takes advantage of the 100% digital transcription of records from the 1880 Census that was organized by the Church of Latter Day Saints and prepared for scholarly use by the Minnesota Population Center. For 39 major cities UTP has added addresses for all residents and is geocoding those addresses based on historical sources. Mapping begins with a contemporary GIS map of Essex County, which required considerable editing (deletion of new roads and other features, insertion of roads that had been eliminated, and correction of street names changed since 1880). In the case of Newark, key resources were a city directory from 1880 that includes address ranges for most streets and a detailed ward map circa 1872 showing the historical street grid. Nearly 97% of addresses have been successfully geocoded.

For the current application, only a portion of the data has been used. First, the main population groups in Newark in 1880 were Germans, Irish and Yankees. Germans and Irish are persons who were born or at least one parent was born in Germany or Ireland, and Yankees are whites born in the U.S. with U.S.-born parents. These groups

comprised about 80% of the population, and for simplicity the analysis only considers these group members. Further only adults age 18 and above are considered. Second, the analysis is limited to the dense central portion of the city. As shown in Figure 2, the study area extends from the downtown area (near the river and including City Hall) and westward into Wards 6 and 13.

Out of a citywide total of 63,390 adult Germans, Irish, and Yankees with geocoded addresses, *N*= 21,520 lived in this portion of Newark. The ethnic composition of the study area was somewhat more German and less Irish than the city as a whole, but all three ethnic groups were well represented. There are *N*=21,520 individuals in the database. In addition to discrete classification of individuals based on ethnicity, with also consider two age categories, namely individuals younger than 30 and older than 30 years of age. There are 7,659 individuals in the dataset classified as Yankees (35.65% of all individuals), of whom 2,667 are <30 years old (34.8% of all Yankees, and 12.40% of all individuals). The most numerous group is German, with 9,450 individuals (43.9% of all cases). Of these, 3,545 are younger than 30 (37.5% of all Germans, and 16.47% of all individuals). Irish are less numerous, with only 4,411 cases classified a belonging to this ethnic group (20.5% of all individuals). Of these, 1,682 are younger than 30 (38.13% of all Irish, and 7.82% of all cases).

In addition to data at the individual level, we also aggregate the information to obtain building occupancy. There are 4,787 unique locations (buildings) that we classify as follows: if the proportion of residents in any one building is greater than 50%, the building is classified as of that ethnic group. If no group is dominant at the 50% level, the building is classified as mixed. Therefore, there are four types of buildings: Irish (323), German (1,710), Yankee (1191), and Mixed (1,563).

Analysis and Results

Clustering and Exposure at the Individual Level

In this section we present the results of the ethnic clustering and exposure analysis. We begin with the more general case of clustering by ethnicity. The results of applying *Q* to the data are summarized in Table 4[1]. The parameters used in the analysis appear there. Using an *m*-surrounding of 5 and overlap degree of 1 (so that any two

[1] MATLAB code to calculate and test *Q* is available as supplementary material that accompanies Ruiz et al. (2010). The code can also be downloaded at:
http://www.science.mcmaster.ca/geo/faculty/paez/publications.html#journals

proximate *m*-surroundings overlap at most in 1 observation), the number of symbolized locations is 5,379[2]. We calculate the statistic using standard and equivalent symbols. The results are highly significant, and indicate that the spatial pattern is not random.

Lack of randomness, as illustrated by the examples in Figure 1, could take different forms. Since the cases are individuals, this could be separation of ethnic groups, or intermingling in the case of the checkerboard pattern. Exploration of the relative frequency of symbols provides additional information about the characteristics of the spatial population pattern. The histogram of the relative frequency of symbols for individuals classified by ethnic group is shown in Figure 3. This figure corresponds to the statistic for equivalent symbols in Table 4. Each bar in the histogram is accompanied by the expected relative frequency of the symbol under the null hypothesis of randomness, and its respective 95% interval of confidence. Recall that the expected value is calculated in consideration to the frequency of members of each ethnic group. Bars for symbols with frequencies that significantly depart from their expected value are color coded: light gray indicates that the frequency exceeds the expectation, and dark gray indicates that the frequency is below the expectation under the null. Several bars in the figure are within the interval of confidence for the symbol. This implies that those symbols do not appear more or less frequently than what would be expected by chance.

Eight symbols appear with significantly more frequency than what would be expected under the null. This includes *m*-surroundings composed exclusively of Germans (005) or Yankees (500), which indicates clustering of these groups. In contrast, Irish do not display a similar tendency towards clustering. Irish, in fact, appear in a cluster as a majority only when in a neighborhood with Yankees. As a minority, they have a tendency to appear more frequently in neighborhoods also with Yankees (see symbols 410 and 320), and to a lesser extent with Germans (see 014). In contrast, mixed *m*-surroundings tend to be rare. All six symbols that appear significantly less frequently are for mixed neighborhoods, and particularly mixed neighborhoods that include Germans (see 113, 122, 203, 212, 221, and 302); the only exception is the case

[2] As noted above, decreasing overlap degree reduces the risk of false positives but also the power of the statistic. The application is therefore very conservative. For thoroughness, we calculated the statistic using r=2, 3, and 4. The statistic is highly significant and rejects the null hypothesis of randomness in every case. As well, the relative frequency of symbols, and their significance, does not display undue variations. Detailed results for this sensitivity analysis are available from the authors.

of a single German in a neighborhood that includes four Yankees (symbol 401). The overall pattern is one of ethnic clustering, especially for Germans and to a lesser extent Yankees. The spatial distribution of Irish individuals is reminiscent of the checkerboard pattern indicative of intermingling, in particular in combination with Yankees.

Patterns of exposure are more easily seen if we reclassify the cases. We illustrate two situations: exposure of Germans, an ethnic group that displays a significant tendency to cluster, to members of other ethnic groups; and exposure of Yankees, almost as numerous as Germans, and with a tendency to cluster that does not match that of Germans. The results of running the statistic for these new classifications (Germans and Others, and Yankees and Others) are shown in Tables 5 and 6. After reclassification, k=2, and the number of symbols is reduced. The statistic is calculated using both standard and equivalent symbols, and the results are, once again, highly significant for both cases.

As before, it is possible to explore the characteristics of the non-random pattern by means of the frequency of symbols. Histograms for the relative frequency of symbols are shown in Figures 4 and 5. In the case of exposure of Germans, all symbols deviate significantly from their expected frequencies. It can be seen in Figure 4 that a frequent occurrence is, in a neighborhood of 5, a given German is exposed solely to other Germans or, at most, one single individual of a different ethnic group (symbols 50 and 41). Likewise, there are significantly more cases of members of other groups not exposed to Germans than what would be expected by chance (symbol 05), although there are also more cases where members of other groups are exposed to a single German (symbol 14).

Clearly, since Germans and Yankees are the two most numerous groups, exposure must be a two-way street between these two groups. Nonetheless, the exposure of Yankees to other Yankees is less marked, in relative terms, than was the case for Germans. And, while there are more cases than expected of members of other groups not exposed to Yankees, again the relative deviation from the expected value is less dramatic. Members of other groups also tend to be less exposed to a single Yankee, however, neighborhoods with three Yankees and two members of other groups occur as one would expect purely by chance.

Sub-classes: Ethnicity and age

Exploration of the spatial distribution of individuals of ethnic groups indicates a tendency towards intra-ethnic clustering, with some mixing between Yankees and Irish. The analysis could be refined by considering additional dimensions, for example marital status, gender, or, as we illustrate in this subsection, age. A new classification scheme now subdivides each ethnic group according to age, those who are younger than 30 (*L30*), and 30 or older (*G30*). Application of the statistic, with m=3 and r=1[3], indicates again that the distribution of individuals by ethnic group and age is not random.

A number of symbols are within the 95% confidence intervals of their expected frequency under the null. Most are significantly more or less frequent than expected. Inspection of the histogram of relative symbol frequencies adds depth to the previous analysis by ethnicity only. For instance, while Germans of all ages tend in general to be in ethnic clusters, this tendency is relatively stronger for older Germans (≥30; see the magnitude of the deviation of symbols 000003 and 000012 from their expected values, compared to symbols 000021 and 000030). The evidence of clustering among Irish was, compared to the other ethnic groups, less strong. In particular, no cluster including 3 Irish individuals was significant in the previous analysis, as seen in Figure 3. When exploring neighborhoods of three, as done here by ethnicity and age, it turns out that older Irish do tend to cluster together (see symbol 000300), but younger Irish do not (see 003000). Of nineteen symbols that appear less frequently than expected, 13 correspond to mixed neighborhoods, with one individual of each ethnic group. Positive deviations from the expected value occur only for mixed group that include Yankees and Irish of different generations. In general, there is more inter-ethnic and inter-generational clustering among Yankees and Irish (see positive deviations for 021000, 020100, and 010200), than among Germans and any other group (see negative deviations for 020010, 010020, 100002, 020001, 100020, and 010002).

Clustering at the Building Level

In our final analysis we show how Q can be applied to a higher level of geography, in this case by aggregating cases to buildings, and classifying buildings as Yankee, Irish, and German, according to the dominant ethnicity, and Mixed if no ethnic

[3] We also calculated the statistic using r=2. The results hold.

group is in the majority. The results of the analysis, using m=4 and r=1[4], appear in Table 8. According to the decision rule, the null hypothesis of a random spatial sequence is rejected, using both standard and equivalent symbols. Further, inspection of the relative symbol frequencies (Figure 7) indicates that mixed buildings tend to be more proximate than expected by chance (see 0004, 1003, and 0103) except when there is a German building in the vicinity (symbol 0013). However, while mixed buildings tend to cluster, mixed clusters of various building types are significantly less common than expected (1111). German buildings are disproportionally more likely to cluster than expected by chance (0040, 0130, 0031) except when there is a Yankee building in the vicinity (1030). This changes when German buildings are not the majority, as clusters of this type are rare (0013, 0022, 1012, and 1021).

Irish buildings, like Irish individuals display less clustering/spatial association. When they do, even accounting for their smaller numbers, they tend to be in more integrated neighborhoods (see 0103 and 0202) or embedded in other ethnic neighborhoods (see 0130 and 3100). Finally, we also find that Yankee buildings also tend to co-locate (see 4000), and when in the majority, they appear with more frequency in company of Irish or mixed buildings (see 3100 and 3001).

Further Opportunities for Spatial Analysis

Symbolization, in addition to forming the basis for statistical analysis as detailed in the preceding sections, also provides the basis for further opportunities for spatial analysis. Having already determined for instance that a certain symbol (e.g. four German buildings in m-surroundings of size 4) appears more (or less) than what would be expected by chance, a question of interest is whether these clusters display a spatial pattern. Figure 8 illustrates this possible use of the symbolized cases. The figure shows in three panels the symbols corresponding to clusters of four Yankee, four German, and four Mixed buildings in m-surroundings of size 4. Clusters of four Irish buildings were not found with greater or lesser frequency than under the null hypothesis of randomness, and are therefore of limited interest. As seen in the figure, clusters of four German buildings display a coherent spatial pattern of concentration along the center and especially north of the study area. In contrast, few clusters of other ethnic buildings are found in the region. Clusters of four mixed buildings are mostly located to the east,

[4] We also calculated the statistic using r=2 and 3. The results hold.

mainly along two or three parallel streets. Clusters or Yankee buildings are mostly in the east and west of the study region, with a tendency towards the southern edge.

Concluding Remarks

In a recent survey of the state of research on ethnic segregation, Kaplan and Woodhouse (2005) reflect on a number of problems that affect traditional approaches to measure segregation, and note progress along different fronts. These issues include the measurement of segregation in situations where multiple groups are present, the fact that many measures do not consider the spatial relationships between units of analysis, and the question of geographical scale. Significant progress has been made in the past few years in terms of addressing some of these issues. In the case of scale, the use of distance-based kernel approaches now allows for the measurement of segregation at multiple scales (e.g. Reardon and O'Sullivan, 2004; Wong, 2004; Feitosa et al, 2007; O'Sullivan and Wong, 2007; Kramer et al, 2010). In terms of spatial relationships between units of analysis, several examples exist of studies that explicitly incorporate them by casting different autocorrelation measures as indicators of segregation (e.g. Brown and Chung, 2006; Johnston et al, 2009; Poulsen et al, 2010). Lastly, there has been progress in the development of measures of multi-group segregation (e.g.Wong, 1998; Reardon and Firebaugh, 2002), and the definition of typologies that seek to refocus analysis on the mix of the population (Johnston et al, 2010).

Use of the *Q* statistic, demonstrated in this paper in an application to historical data, follows on the heels of some of these advances, and augments the analytical possibilities in segregation research. Our approach has a number of qualities to recommend it, indeed some that contribute positively to several of the issues identified by Kaplan and Woodhouse in their survey. First, the *Q* statistic can naturally accommodate multiple groups, as illustrated in our analysis of three different population classes (Irish, Germans, and Yankees) and even subclasses (single or married). Secondly, *Q* is inherently about spatial relationships between units of analysis, which enter the statistic by means of the definition of *m*-surroundings, or neighborhoods of size *m*. This leads to the first noteworthy aspect of our approach with respect to scale. It is known that when *Q* detects association of a spatial qualitative variable at the level of *m*, the association carries down to subsets of size smaller than *m* (Ruiz et al, 2010; pp. 290-291). For this reason, selection of *m* allows the analyst to explore spatial association at different scales, in the confidence that results are consistent for smaller

scales. It must be noted, though, that interpretability of the results may become an issue at larger *m*-surrounding sizes, as the number of symbols increases. The second issue related to scale is that, unlike other approaches that are based on proportions and/or densities and that are therefore inherently areal, the *Q* statistic can be applied at the most basic level of analysis (the individual), and can be scaled up as desired, as illustrated in our analysis at the level of building occupancy. An intriguing possibility in terms of scaling up the analysis to administrative areas, is to combine the spatial dimension of *Q* with the (inherently categorical) typology of population mixes of Johnston et al. (2007). This, we suggest, is a worthy avenue for future research. As well, comparing Q to other existing measures of clustering and exposure may provide additional insights into the appropriateness of various measures in different application contexts.

On a final note, our approach does not aspire to be general (q.v. Reardon and O'Sullivan, 2004; Wong, 2005). Rather, we would argue that its value resides precisely in its specificity, since it unambiguously deals with only one dimension of segregation, on the clustering-exposure continuum. That being said, we suggest that a more complete spatial analytical framework to explore segregation could combine our approach to measuring clustering-exposure, and the use of local indicators of spatial association (Anselin, 1995) or concentration (Getis and Ord, 1992) as measures of concentration-eveness. This would provide a fully spatial picture of the two major dimensions of segregation.

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Table 1. List of symbols for k=2 and m=4

σ_1 ={0,0,0,0}	σ_5 ={1,0,0,0}	σ_9 ={0,1,0,1}	σ_{13} ={0,1,1,1}
σ_2 ={0,0,0,1}	σ_6 ={0,0,1,1}	σ_{10} ={1,0,1,0}	σ_{14} ={1,1,1,0}
σ_3 ={0,0,1,0}	σ_7 ={0,1,1,0}	σ_{11} ={1,0,0,1}	σ_{15} ={1,1,0,1}
σ_4 ={0,1,0,0}	σ_8 ={1,1,0,0}	σ_{12} ={1,0,1,1}	σ_{16} ={1,1,1,1}

Table 2.Frequency and relative frequency of symbols in example. In the frequency, each I indicates one occurrence of the symbol in the diagram.

	Diagram 1		Diagram 2		Diagram 3	
	n_{σ_j}	p_{σ_j}	n_{σ_j}	p_{σ_j}	n_{σ_j}	p_{σ_j}
σ_1 ={0,0,0,0}	IIII IIII II	0.333		0.000	II	0.056
σ_2 ={0,0,0,1}		0.000		0.000	I	0.028
σ_3 ={0,0,1,0}		0.000		0.000	IIII	0.111
σ_4 ={0,1,0,0}	IIII I	0.167		0.000	II	0.056
σ_5 ={1,0,0,0}		0.000	IIII IIIIIIII I	0.444	II	0.056
σ_6 ={0,0,1,1}		0.000		0.000	II	0.056
σ_7 ={0,1,1,0}		0.000	II	0.056	I	0.028
σ_8 ={1,1,0,0}		0.000		0.000	II	0.056
σ_9 ={0,1,0,1}		0.000		0.000	III	0.083
σ_{10} ={1,0,1,0}		0.000		0.000	III	0.083
σ_{11} ={1,0,0,1}		0.000	II	0.056	IIII	0.111
σ_{12} ={1,0,1,1}		0.000		0.000		0.000
σ_{13} ={0,1,1,1}		0.000	IIII IIIIIIII I	0.444	III	0.083
σ_{14} ={1,1,1,0}	IIII	0.139		0.000	III	0.083
σ_{15} ={1,1,0,1}	I	0.028		0.000	II	0.056
σ_{16} ={1,1,1,1}	IIII IIII II	0.333		0.000	II	0.056

Table 3.Equivalent symbols for k=2 and m=4.

Equivalent Symbol	Standard Symbols
σ^*_1 ={4,0}	{0,0,0,0}
σ^*_2 ={3,1}	{0,0,0,1} , {0,0,1,0} , {0,1,0,0} , {1,0,0,0}
σ^*_3 ={2,2}	{0,0,1,1} , {0,1,1,0} , {1,1,0,0} , {1,0,0,1} , {0,1,0,1} , {1,0,1,0} , {1,0,0,1}
σ^*_4 ={1,3}	{1,0,1,1} , {0,1,1,1} , {1,1,1,0} , {1,1,0,1}
σ^*_5 ={0,4}	{1,1,1,1}

Table 4. Clustering by ethnicity

Number of cases N	21,520		
Symbolized locations S	5,379		
Number of classes k	3		
Size of m-surrounding	5		
Degree of overlap r	1		
Number of standard symbols (σ)	243		
Number of equivalent symbols (σ^*)	21		
Frequency of classes	Y: 0.3559	I: 0.2050	G: 0.4391
Spatial Association Test	Statistic	Degrees of Freedom	p-value
$Q(5)$ (standard symbols)	2,276.89	242	0.0000
$Q(5)$ (equivalent symbols)	2,050.01	20	0.0000

Table 5. Exposure Germans

Number of cases N	21,520		
Symbolized locations S	5,379		
Number of classes k	2		
Size of m-surrounding	5		
Degree of overlap r	1		
Number of standard symbols (σ)	32		
Number of equivalent symbols (σ^*)	6		
Frequency of classes	G: 0.4391	O: 0.5609	
Spatial Association Test	Statistic	Degrees of Freedom	p-value
$Q(5)$ (standard symbols)	1,792.84	31	0.0000
$Q(5)$ (equivalent symbols)	1,774.23	5	0.0000

Table 6. Exposure Yankees

Number of cases N	21,520		
Symbolized locations S	5,379		
Number of classes k	2		
Size of m-surrounding	5		
Degree of overlap r	1		
Number of standard symbols (σ)	32		
Number of equivalent symbols (σ^*)	6		
Frequency of classes	Y: 0.3559	O: 0.6441	
Spatial Association Test	Statistic	Degrees of Freedom	p-value
$Q(5)$ (standard symbols)	1,138.00	31	0.0000
$Q(5)$ (equivalent symbols)	1,121.54	5	0.0000

Table 7.Clustering by ethnicity and age

Number of cases N	21,520		
Number of symbolized locations S	10,759		
Number of classes k	6		
Size of m-surrounding	3		
Degree of overlap r	1		
Number of standard symbols (σ)	216		
Number of equivalent symbols (σ^*)	56		
Frequency of classes	*YL30*: 0.1239	*IL30*: 0.0782	*GL30*: 0.1647
	YG30: 0.2320	*IG30*: 0.1268	*GG30*: 0.2744
Spatial Association Test	Statistic	Degrees of Freedom	p-value
$Q(3)$ (standard symbols)	1,667.45	215	0.0000
$Q(3)$ (equivalent symbols)	1,482.29	55	0.0000

Table 8. Clustering by building ethnicity

Number of cases N	4,787		
Number of symbolized locations S	1,595		
Number of classes k	4		
Size of m-surrounding	4		
Degree of overlap r	1		
Number of standard symbols (σ)	256		
Number of equivalent symbols (σ^*)	35		
Frequency of classes	*Y*: 0.2488	*I*: 0.0675	
	G: 0.3572	*M*: 0.3265	
Spatial Association Test	Statistic	Degrees of Freedom	p-value
$Q(4)$ (standard symbols)	1,503.82	255	0.0000
$Q(4)$ (equivalent symbols)	1,231.25	34	0.0000

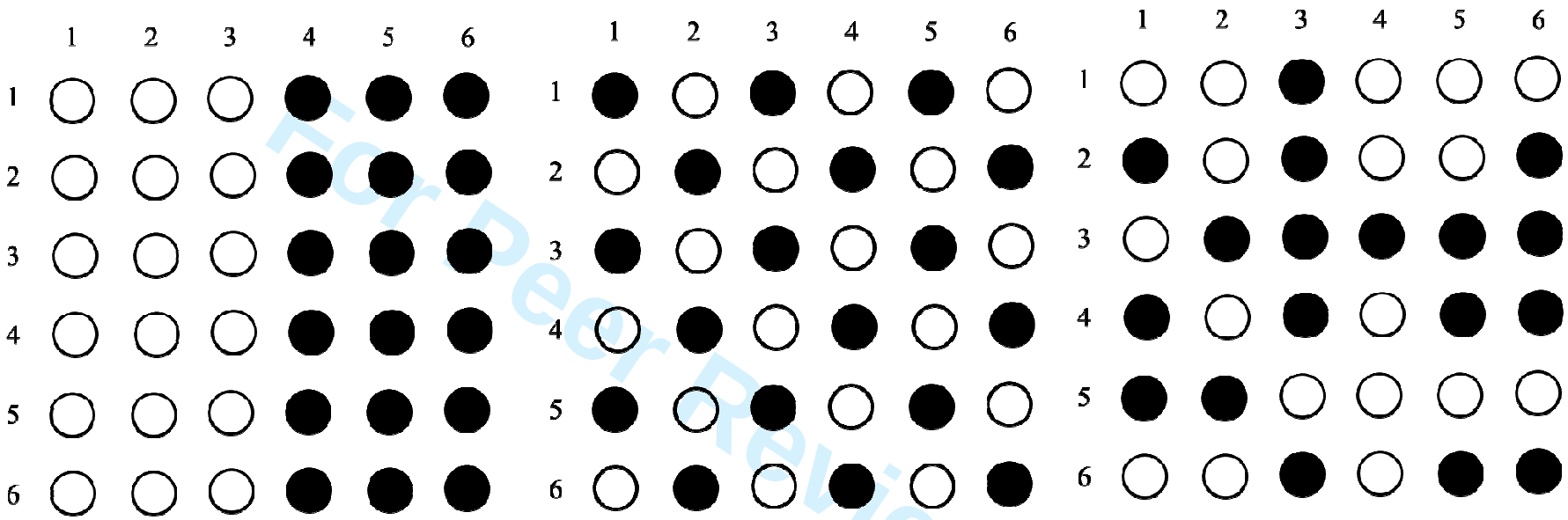

Figure 1. Examples of non-random and random spatial black-and-white patterns

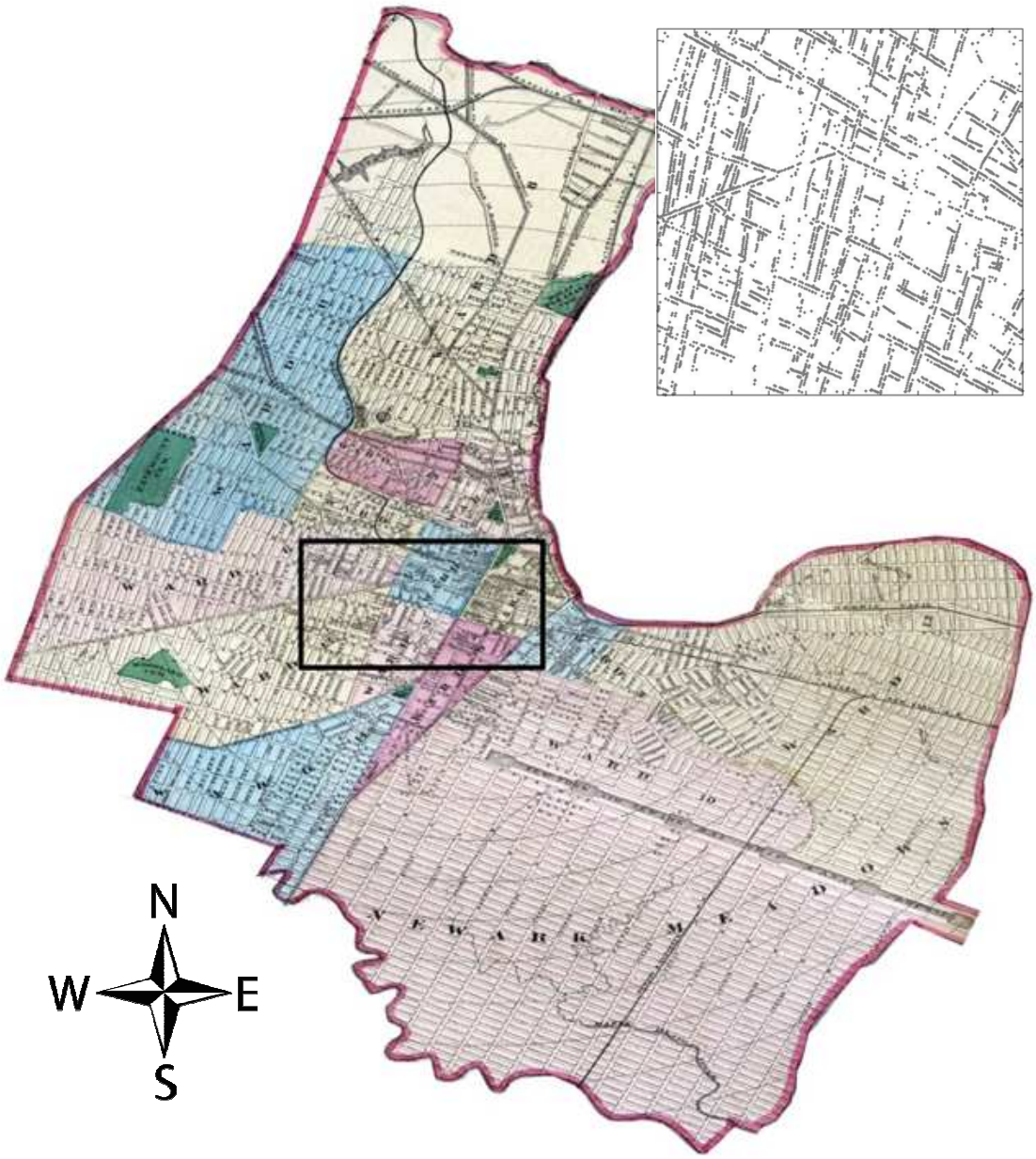

Figure 2. The City of Newark, NJ, in 1880, showing the approximate boundaries of the study area. The spatial distribution of cases appears in the inset.

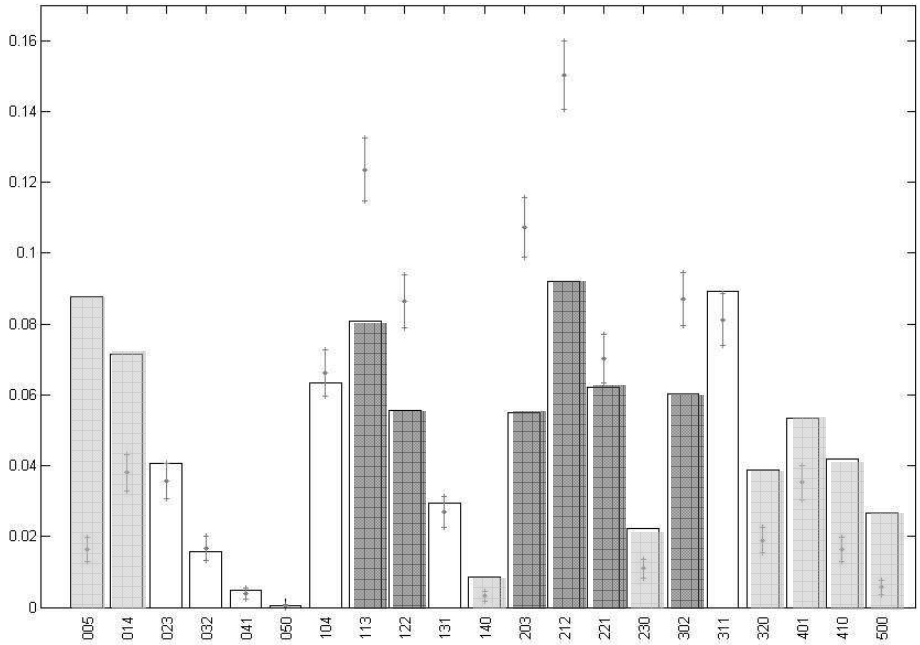

Figure 3. Relative symbol frequency for $Q(5)$ and ethnic classes Yankee, Irish, German. The sequence of numbers on the x axis denote $n_Y\ n_I\ n_G$ in an m-surrounding of 5 (e.g. 113 is 1 Yankee, 1 Irish, and 3 Germans).

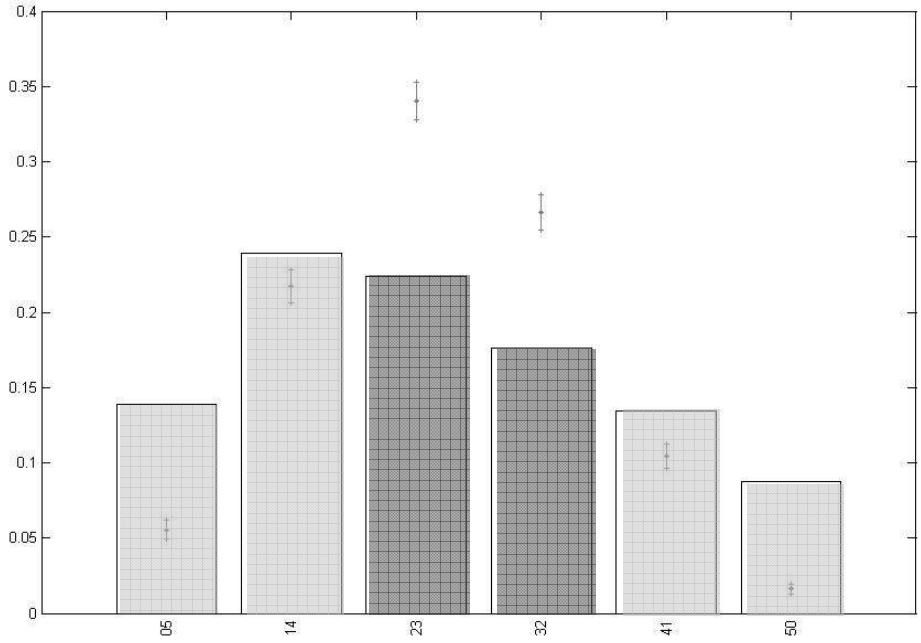

Figure 4. Relative symbol frequency for $Q(5)$ and classes German and Other. The sequence of numbers on the x axis denote $n_G\ n_O$ in an m-surrounding of 5 (e.g. 23 is 2 Germans, 3 Others).

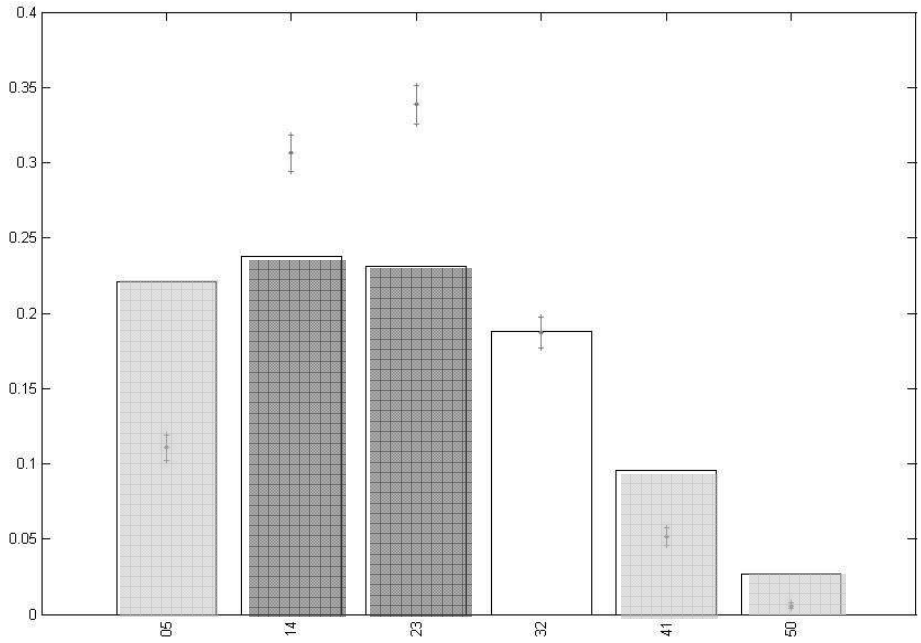

Figure 5. Relative symbol frequency for $Q(5)$ and classes Yankee and Other. The sequence of numbers on the x axis denote n_G n_O in an m-surrounding of 5 (e.g. 23 is 2 Germans, 3 Others).

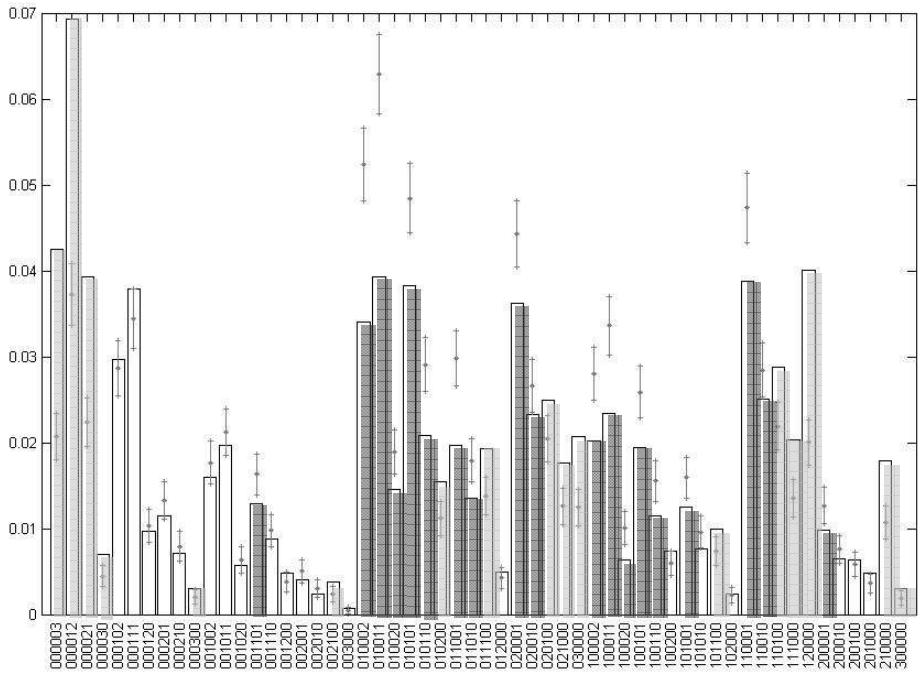

Figure 6. Relative symbol frequency for $Q(3)$ and classes Yankee (L30 and G30), Irish (L30 and G30), and German (L30 and G30). The sequence of numbers on the x axis denote n_{YL30} n_{YG30} n_{IL30} n_{IG30} n_{GL30} n_{GG30} in an m-surrounding of 3 (e.g. 000003 is 3 Germans, 30 years or older).

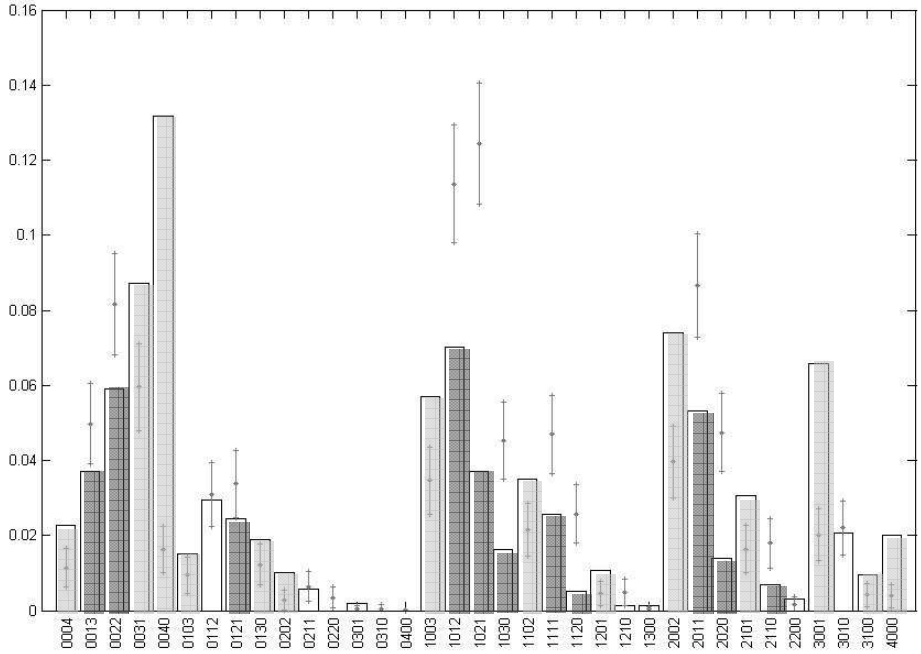

Figure 7. Relative symbol frequency for $Q(4)$ and building classes Yankee, Irish, German, and Mixed. The sequence of numbers on the x axis denote $n_Y\, n_I\, n_G\, n_M$ in an m-surrounding of 4 (e.g. 1012 is 1 Yankee, 0 Irish, 1 German, 2 Mixed).

Figure 8. Spatial distribution of symbolized cases: (◊) four Yankee buildings in *m*-surrounding (*m*=4); (○) four German buildings in *m*-surrounding (*m*=4); (□) four Mixed buildings in *m*-surrounding (*m*=4).